Natural convection to power-law fluids from a heated vertical plate in a stratified environment

Jin Kook Lee
Engineering Technology, Aircraft Accessories, Argo-Tech Corporation, Cleveland, OH, USA
Rama Subba Reddy Gorla
Department of Mechanical Engineering, Cleveland State University, Cleveland, OH, USA
Ioan Pop
Faculty of Mathematics, University of Cluj, Cluj, Romania

A theoretical analysis is presented that brings steady, laminar free convection from a vertical flat plate immersed in a power-law stratified fluid within the framework of classical boundary-layer theory. Two classes of similarity solutions corresponding to linearly increasing (positive *M* class) and linearly decreasing (negative *M* class) plate temperature with the plate height are studied. These similarity solutions are numerically determined for the case of fixed wall temperature and variable environment temperature, as well as for the case of fixed environment temperature with variable wall temperature. Of special interest are the effects of the power-law viscosity index, the generalized Prandtl number, the stratification parameter, and the type of thermal wall condition on the velocity and temperature fields and hence on the skin-friction coefficient and the wall heat transfer parameter.

Keywords: natural convection; non-Newtonian fluids; rheology; power-law fluids

Introduction

Of particular interest in classical boundary-layer theory are those fluid-flow problems for which the governing equations possess similarity solutions. The problem of natural convective heat transfer from bodies immersed in fluids does not always admit similarity solutions. The nonsimilarity in this case arises either due to thermally stratified fluid (in which the body is immersed) or due to nonuniform surface temperature of the body.

Recently, new classes of similarity solutions for a natural convective flow of a Newtonian fluid on a vertical flat plate suspended in a quiescent, thermally stratified medium have been reported by Semenov (1984), Kulkarni et al. (1987), and Henks and Hoogendoorn (1989). Stratified fluid occurs quite frequently in nature (for example, in the atmosphere and ocean), in heat transfer in closed containers, in environmental chambers, in several heat-rejection and energy-storage processes, and in a number of engineering devices. In fact, this subject is not a new one, for it has been taken up for consideration from the beginning of experimental studies on natural convection in liquids; detailed references of the earlier work can be found in the papers by Takeuchi et al. (1975), Chen and Eichhorn (1976), and Jaluria and Himasekhar (1983). The problem has been also recently extended by Bejan (1984), Takhar and Pop (1987), Nakayama and Koyama (1987) to the case of natural convection from a heated surface embedded in a thermally stratified porous medium.

Address reprint requests to Professor Gorla at Cleveland State University, Cleveland, OH 44115, USA.

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A number of industrially important fluids such as molten plastics, polymers, pulps, foods, etc. exhibit non-Newtonian fluid behavior. Due to the growing use of these non-Newtonian substances in various manufacturing and processing industries, considerable efforts have been directed towards understanding their friction and heat transfer characteristics. However, many of the inelastic non-Newtonian fluids encountered in chemical engineering processes are known to follow the power-law model in which the shear stress varies according to a power function of the strain rate. But for this relatively simple power-law model, the mathematical complexity of the governing equations increases because of the extra nonlinearity in the viscous terms. A fundamental question then arises: Do the governing equations for the natural convection from a vertical plate immersed in stratified power-law fluids possess similarity solutions?

The objective of this study is to show that the elegant similarity solutions devised by Semenov (1984) can be extended to the more general case of power-law fluids. However, for the sake of simplicity, we consider only the situation when both the temperatures of the plate $T_w(x)$ and of the ambient fluid $T_\infty(x)$ vary linearly with the distance x from the leading edge. Thus, we will obtain a class of similarity solutions for this problem that can be reduced to the proper solution T_w = constant and $T_\infty(x)$ linearly decreasing with x as well as to $T_w(x)$ linearly increasing with x and T_∞ = constant. Discussions are concentrated on how the velocity and temperature profiles and the skin friction and heat transfer coefficients in the boundary layer are influenced by the flow behavior index n, the generalized Prandtl number Pr, and the parameter m indicating whether the ambient temperature or the wall temperature is fixed.

Analysis

Consider a natural-convection laminar boundary-layer flow along a vertical semi-infinite flat plate immersed in a thermally stratified, quiescent power-law fluid with the following transport properties as given by Shulmann and Berkovsky (1966), Shvets and Vishneski (1987), Gryglaszewski and Saljnikov (1989), Pop and Gorla (1990), and Pop et al. (1991) in the following manner:

$$\tau_{ij} = K|\tfrac{1}{2}I_2|^{(n-1)/2}e_{ij} \tag{1}$$

$$q = -k|\tfrac{1}{2}I_2|^{s/2}\,\text{grad}\,T \tag{2}$$

where τ_{ij} and e_{ij} are the tensors of stress and strain-rate, respectively, I_2 is the second invariant of the strain-rate tensor, T is the temperature inside the boundary layer, K and k are the consistency index and thermal conductivity, respectively, and n and s are the exponents identifying non-Newtonian behavior in the flow and heat transfer, respectively. The strict Boussinesq approximation is assumed, i.e., the variation of fluid density with temperature is accounted for only in the buoyancy term of the momentum equation; all other fluid properties are assumed to be constant, and viscous dissipation is neglected. We choose (x, y) as coordinates, with the x-axis measured along the plate in the upward direction and the y-axis normal to it. Under these considerations, the steady laminar boundary-layer equations can be written as given by Shenoy and Mashelkar (1982) and Wang and Kleinstreuer (1987) in the following manner:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{s}\frac{\partial T}{\partial y}\right) \tag{5}$$

which are to be solved with boundary conditions

$$\begin{aligned} y = 0:&\quad u = v = 0, \quad T = T_w(x) \\ y = \infty:&\quad u = 0, \quad T_\infty(x) \end{aligned} \tag{6}$$

In these equations, (u, v) are the velocity components along the (x, y)-axes, g is the acceleration due to gravity, β is the coefficient of thermal expansion, and ρ and α are the density and thermal diffusivity, respectively.

For spatial distributions $T_w(x)$ and $T_\infty(x)$, a similarity solution of Equations 3 to 6 exists. Thus, the temperature is written as

$$T = (m + h(\xi, \eta))\xi\,\Delta T + T_r \tag{7}$$

with

$$\begin{aligned} \eta = 0:&\quad h(\xi,\eta) = 1, \quad T_w = (m+1)\xi\,\Delta T + T_r \\ \eta \to \infty:&\quad h(\xi,\eta) = 0, \quad T_\infty = m\xi\,\Delta T + T_r \end{aligned} \tag{8}$$

where h is the reduced temperature function, $\Delta T = T_w(0) - T_\infty(0)$ is a characteristic temperature difference, and T_r is a constant. The transformed coordinates in Equation 7 are

$$\xi = Mx + N \ (\xi \geqslant 0)$$

$$\eta = [(g\beta\,\Delta T)^{2-n}(\rho/k)^2|M|^n]^{1/2(n+1)}\xi^{(1-n)/(1+n)}y$$

M and N being constants. We shall further see that the transformed x-coordinate defines two classes of similarity solutions that can be completely characterized by the expression $\text{sgn}(M)$. These similarity solutions are called the positive M class if $\text{sgn}(M) = 1$ at $M > 0$ and the negative M class if $\text{sgn}(M) = -1$ at $M < 0$, respectively (see Henkes and Hoogendoorn 1989).

Next, the stream function ψ is introduced of the form

$$\psi = \left[\frac{(g\beta\,\Delta T)^{2n-1}}{|M|^{2n+1}}(K/\rho)^2\right]^{1/2(n+1)}\xi^{2n/(n+1)}f(\xi,\eta)$$

which defines the $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ velocity components as

$$u = (g\beta\,\Delta T/|M|)^{1/2}\xi f'$$

$$v = -M\left[\frac{(g\beta\,\Delta T)^{2n-1}}{|M|^{2n+1}}(K/\rho)^2\right]^{1/2(n+1)}\xi^{(n-1)/(n+1)} \times \left(\frac{2n}{n+1}f + \frac{1-n}{1+n}\eta f' + \xi\frac{\partial f}{\partial \xi}\right) \tag{9}$$

Notation

e_{ij}	Strain-rate tensor
f	Reduced stream function
g	Acceleration due to gravity
h	Reduced temperature function
I_2	Second invariant of the strain-rate tensor
K	Consistency index
k	Thermal conductivity
m	Parameter describing whether the environment temperature ($m = 0$) or the wall temperature ($m = -1$) is fixed
M, N	Coefficients in the ξ-coordinate
n	Flow behavior index
Pr	Generalized Prandtl number
s	Heat transfer behavior index
T	Temperature
T_r	Reference temperature
ΔT	Characteristic temperature difference
u, v	Velocity components along (x, y)-axes
x, y	Coordinates along and normal to the plate

Greek symbols

α	Thermal diffusivity
β	Thermal expansion coefficient
δ	Unit tensor
η	Similarity y-coordinate
ξ	Transformed x-coordinate
ρ	Density
τ	Stress tensor
ψ	Stream function

Superscripts

$'$	Differentiation with respect to η

Subscripts

w	Wall condition
∞	Environment condition

Here, primes denote differentiation with respect to η. Equations 4 and 5 now reduce to

$$(|f''|^{n-1}f'')' + \mathrm{sgn}(M)\left[\frac{2n}{n+1}ff'' - (f')^2\right] + h = \mathrm{sgn}(M)\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \quad (10)$$

$$\frac{1}{\mathrm{Pr}}\xi^{2(1-n+s)/(1+n)}(|f''|^s h')' + \mathrm{sgn}(M)\left[\frac{2n}{n+1}fh' - (h+m)f'\right] = \mathrm{sgn}(M)\left(f'\frac{\partial h}{\partial \xi} - h'\frac{\partial f}{\partial \xi}\right) \quad (11)$$

and boundary conditions 6 and 8 become

$$\eta = 0: \quad 2nf + (n+1)\xi\frac{\partial f}{\partial \xi} = 0, \quad f' = 0, h = 1$$
$$\eta \to \infty: \quad f' = 0, \quad h = 0 \quad (12)$$

In Equation 11, Pr is the generalized Prandtl number given by

$$\mathrm{Pr} = \frac{1}{\alpha}[(g\beta\,\Delta T)^3/|M|]^{(n-1-s)/2(n+1)}(K/\rho)^{(s+n)/(n+1)}$$

We see from Equations 10 to 12 that they become similar when

$$s = n - 1$$

The mathematical problem defined by Equations 10 to 12 then transforms into

$$(|f''|^{n-1}f'')' + \mathrm{sgn}(M)\left[\frac{2n}{n+1}ff'' - (f')^2\right] + h = 0 \quad (13)$$

$$\frac{1}{\mathrm{Pr}}(|f''|^{n-1}h')' + \mathrm{sgn}(M)\left[\frac{2n}{n+1}fh' - (h+m)f'\right] = 0 \quad (14)$$

and

$$f(0) = f'(0) = 0, \quad h(0) = 1$$
$$f'(\infty) = h(\infty) = 0 \quad (15)$$

The above is a set of coupled, nonlinear, second-order, ordinary differential equations with linear boundary conditions that do not contain any function of x. Note that Equations 13 and 14 for $n = 1$ describe the free-convection flow from a vertical plate immersed in a Newtonian stratified medium. The deviation of n from unity indicates the degree of deviation from Newtonian behavior.

Special situations in Equations 13 and 14 are $m = 0$ for the non-stratified environment and $m = -1$ for the fixed wall-temperature situation. The environment is stably stratified if $dT_\infty/dx > 0$, and hence $mM > 0$. Then, we notice that the dependence of the equations on the $\mathrm{sgn}(M)$ reflects the fact that convection depends substantially on whether the temperature of the plate increases ($dT_w/dx > 0$) or decreases ($dT_w/dx < 0$) along the height. No solutions were presented for Equations 13 and 15 before, and these solutions are reported here for the first time.

Of particular interest in most boundary-layer natural-convection problems are the velocity and temperature gradients at the wall, from which the local skin-friction and local heat transfer coefficients can be calculated. Thus, the skin friction and heat transfer at the wall can be expressed as

$$T_w = K[(g\beta\,\Delta T)^3(\rho/K)^2\xi^4/|M|]|f''(0)|^n \quad (16)$$

$$q_w = k\,\Delta T[(g\beta\,\Delta T)^{2n-1}(\rho/K)^{2n}|M|\xi^4]^{1/2(n+1)} \times |f''(0)|^{n-1}h'(0) \quad (17)$$

which make $f''(0)$ and $h'(0)$ important parts of the solution.

Numerical solution

The numerical procedure used here solves the two-point boundary-value problems for a system of N ordinary differential equations in the range (X, X_1). The system is written as

$$\frac{dy_i}{dx} = f_i(x, y_1, y_2, \ldots, y_N), \quad i = 1, 2, 3, \ldots, N$$

and the derivatives f_i are evaluated by a procedure that evaluates the derivatives of $y_1, y_2, \ldots, y_N$ at a general point X. Initially, N boundary values of the variable y_i must be specified, some of which will be specified at X and some at X_1. The remaining N boundary values are guessed, and the procedure corrects them by a form of Newton iteration. Starting from the known and guessed values of y_i at X, the procedure integrates the equations forward to a matching point R, using Merson's method. Similarly, starting from X_1 it integrates backwards to R. The difference between the forward and backward values of y_i at R should be zero for a true solution. The procedure uses a generalized Newton method to reduce these differences to zero by calculating corrections to the estimated boundary values. This process is repeated iteratively until convergence is obtained to a specified accuracy.

The tests for convergence and the perturbation of the boundary conditions are carried out in a mixed form. For example, if the error estimate for y_i is ERROR_i, we test whether $\mathrm{ABS}\,(\mathrm{ERROR}_i) < \mathrm{ERROR}_i \times (1 + \mathrm{ABS}\; y_i)$. Essentially, this makes the test absolute for $y_i \ll 1$ and relative for $y_i \gg 1$. Note that convergence is not guaranteed, particularly from a poor starting approximation.

A serious difficulty that may arise with boundary-value problems is inherent instability. In such cases, integration from one or both ends of the range will produce rapidly increasing solutions that may occasionally lead to overflow before the matching point is reached. The position of the matching point R can be varied to improve the situation; if the solution increases rapidly for forward (or backward) integration, R should be taken at X (or X_1); if it increases in both directions, R should be taken between X and X_1. If the matching point R is at one of the endpoints X or X_1, there is no need to estimate the unknown boundary values accurately, since there they are not required for integration. Another difficulty that often arises is the case when one end of the range, say X_1, is at infinity. The end-point is approximated by taking finite values for X_1, which is obtained by estimating where the solution will reach its asymptotic state. The computing time for integrating the differential equations can sometimes depend critically on the quality of the initial guesses of the unknown boundary conditions, the locations of the matching point and the infinite endpoint.

Results and discussion

Solutions of Equations 13 to 15 have been determined for some values of the power-law index ranging from $n = 0.5$ to 1.5 and Pr = 10 and 100, respectively.

The following cases are considered: case I, $m = 0$, $\mathrm{sgn}(M) = 1$, (nonstratified environment with increasing wall temperature); case II, $m = -1$, $\mathrm{sgn}(M) = 1$ (unstably stratified environment with fixed wall temperature); case III, $m = 0$, $\mathrm{sgn}(M) = -1$ (nonstratified environment with decreasing wall temperature); and case IV, $m = -1$, $\mathrm{sgn}(M) = -1$ (stably stratified environment with fixed wall temperature).

The resulting similarity solutions for the reduced velocity $f'(\eta)$ and temperature $h(\eta)$ profiles are displayed in Figures 1 through 8 for three power-law fluids: $n = 0.5$ (pseudoplastic),

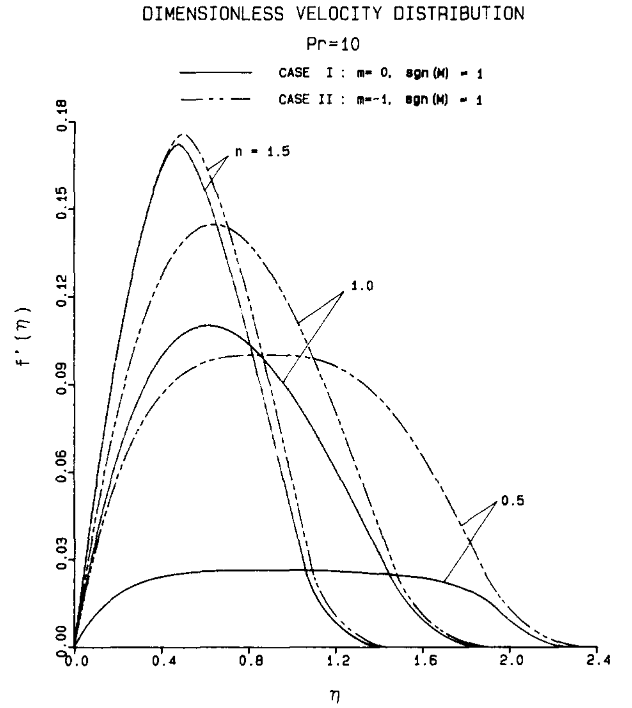

Figure 1 Dimensionless velocity distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 10$); for case I: $m = 0$; $\mathrm{sgn}(M) = 1$; case II: $m = -1$, $\mathrm{sgn}(M) = 1$

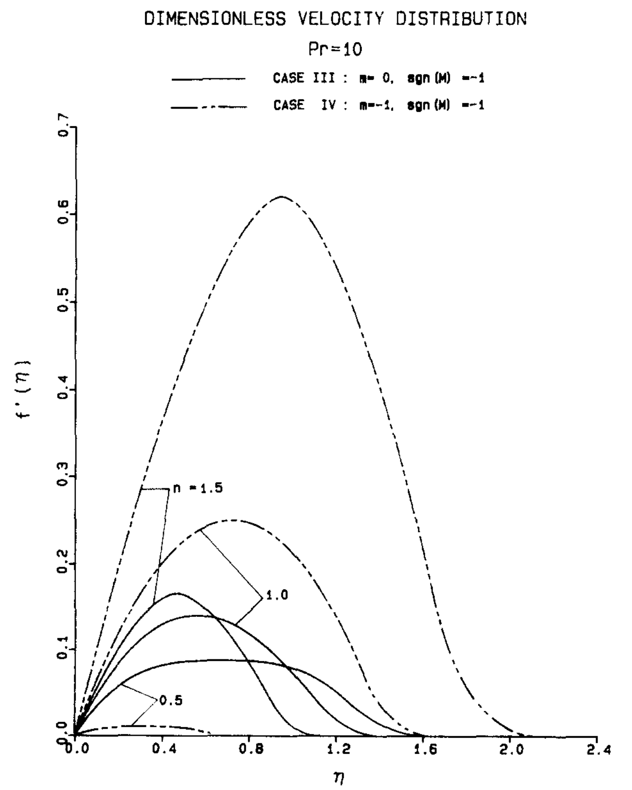

Figure 2 Dimensionless velocity distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 10$): for case III: $m = 0$, $\mathrm{sgn}(M) = -1$; case IV: $m = -1$, $\mathrm{sgn}(M) = -1$

$n = 1$ (Newtonian), and $n = 1.5$ (dilatant). Figures 1 to 4 show that the momentum boundary-layer thickness decreases as the flow behavior index n increases for the positive M-class solutions. For the nonstratified environment case and negative M-class solutions, the momentum boundary-layer thickness decreases as n increases. However, for the fixed wall-temperature case ($m = 1$), the opposite behavior is observed. The velocity profiles are lower for $\mathrm{sgn}(M) = 1$ than those for $\mathrm{sgn}(M) = -1$. They also decrease with the increase of the generalized Prandtl number.

Figures 5 to 8 depict the effects of the system parameters on the temperature distribution within the boundary layer. The difference in $h(\eta)$ resulting from the two positive and negative M-class similarity solutions is clearly seen here. These figures would also indicate that there is a temperature reversal in the boundary layer that may be stronger at high Pr and weaker at lower Pr. The reversal in the temperature occurs because the cooler fluid from the bottom overshoots upward to a level where the ambient temperature is higher. We note that for a Newtonian fluid ($n = 1$), the flow- and temperature-reversal regimes are predicted by Kulkarni et al. (1987) and by Henkes and Hoogendoorn (1989). Thus there appears to be good qualitative agreement between the present results when $n = 1$ and the literature values. It is quite likely that the power-law index n has a profound effect on the magnitude of the temperature reversal. We hope to be able to report on this matter in due course.

Furthermore, we see from Figure 7 that for a nonstratified environment ($m = 0$) there is initially a fall of the temperature profile within the boundary layer from its value at the wall. This fall is then followed by an increase of the temperature profile. The peak temperature for a dilatant fluid ($n = 1.5$) has the largest value when compared with the peak magnitude for a Newtonian fluid ($n = 1$). Such a behavior was not observed at lower values of the Prandtl number.

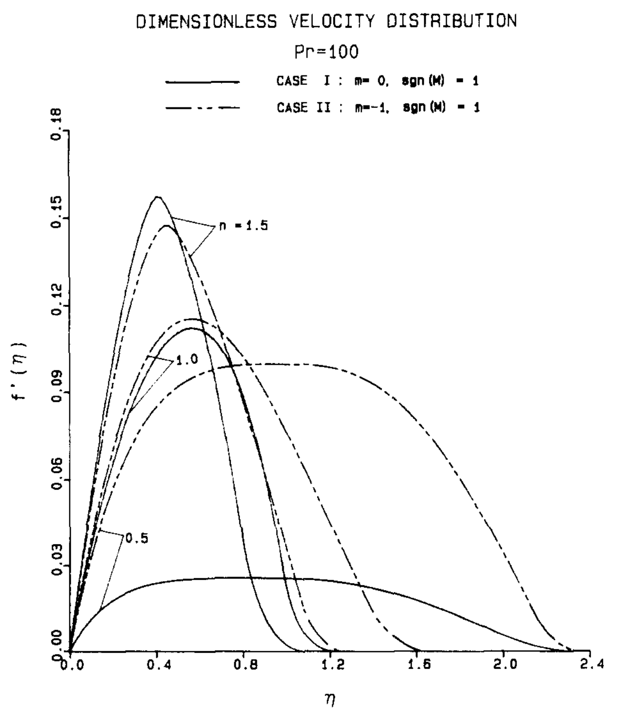

Figure 3 Dimensionless velocity distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 100$): for case I: $m = 0$, $\mathrm{sgn}(M) = 1$; case II: $m = -1$, $\mathrm{sgn}(M) = 1$

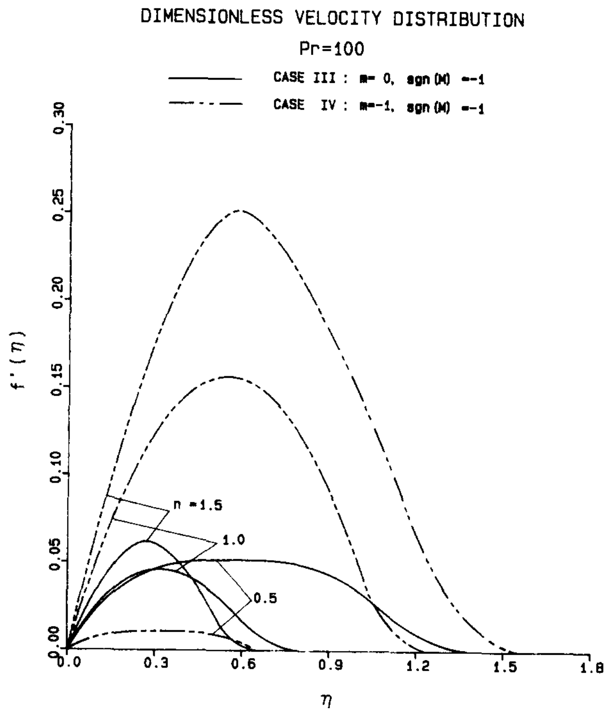

Figure 4 Dimensionless velocity distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 100$): for case III: $m = 0$, $\mathrm{sgn}(M) = -1$; case IV: $m = -1$, $\mathrm{sgn}(M) = -1$

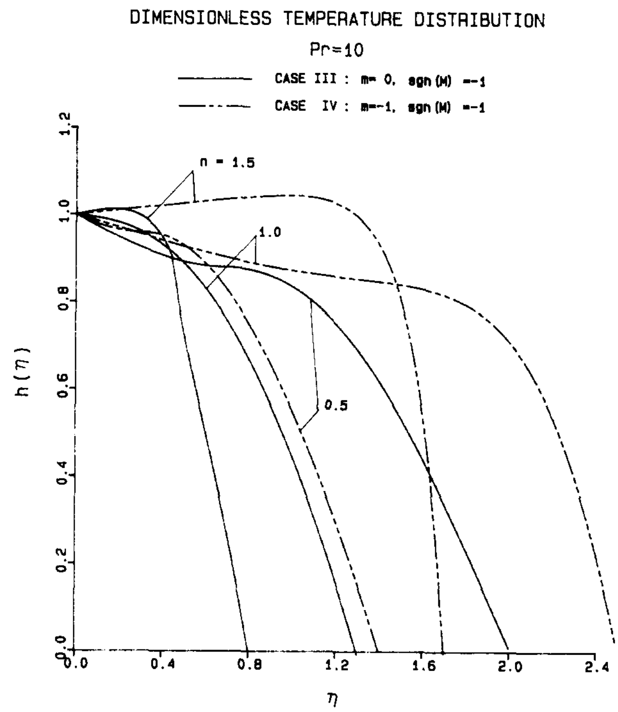

Figure 6 Dimensionless temperature distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 10$): for case III: $m = 0$, $\mathrm{sgn}(M) = -1$; case IV: $m = -1$, $\mathrm{sgn}(M) = -1$

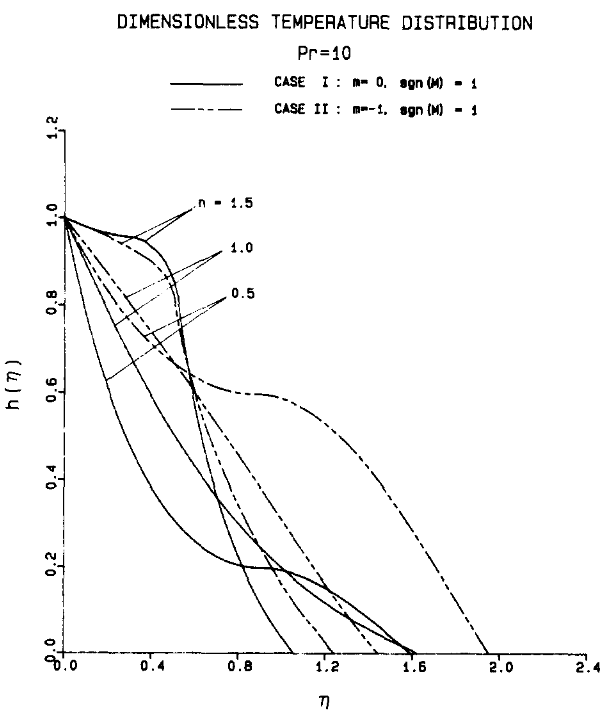

Figure 5 Dimensionless temperature distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 10$): for case I: $m = 0$, $\mathrm{sgn}(M) = 1$; case II: $m = -1$, $\mathrm{sgn}(M) = 1$

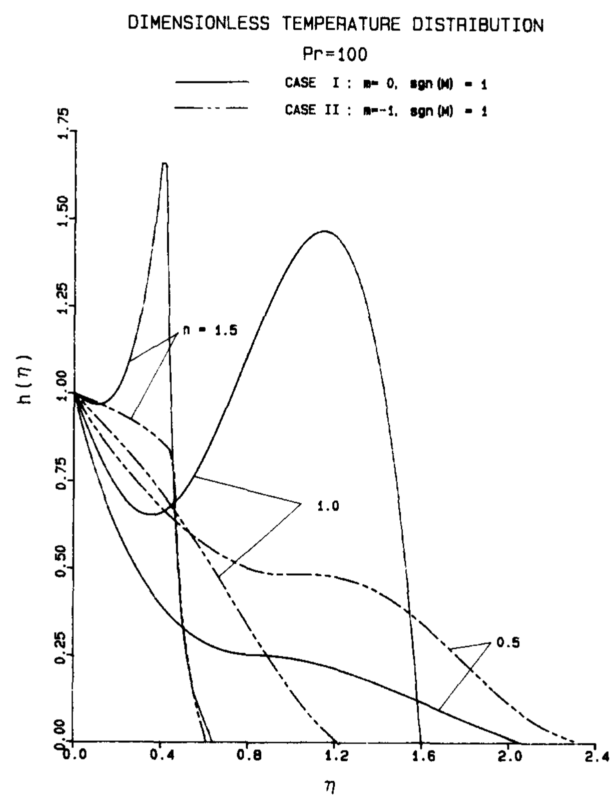

Figure 7 Dimensionless temperature distribution *versus* similarity variable η for various flow behavior indexes n ($\mathrm{Pr} = 100$): for case I: $m = 0$, $\mathrm{sgn}(M) = 1$; case II: $m = -1$, $\mathrm{sgn}(M) = 1$

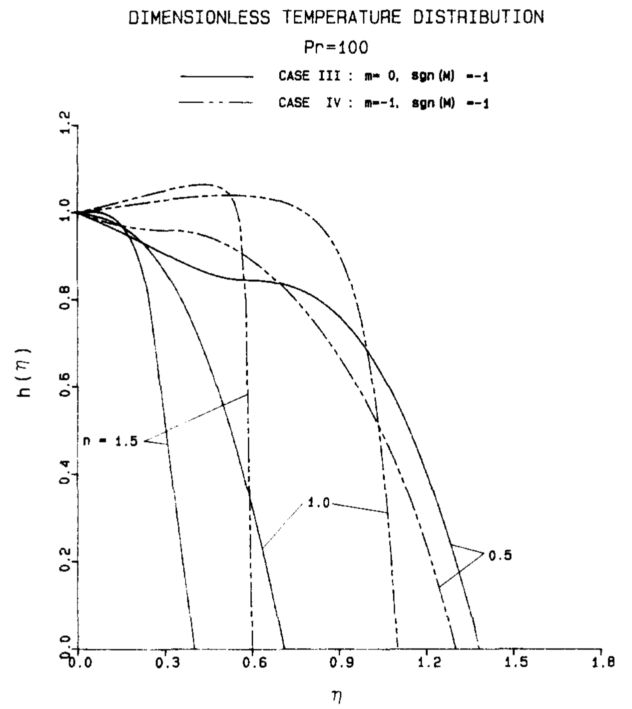

Figure 8 Dimensionless temperature distribution *versus* similarity variable η for various flow behavior indexes n ($Pr = 100$): for case III: $m = 0$, $sgn(M) = -1$; case IV: $m = -1$, $sgn(M) = -1$

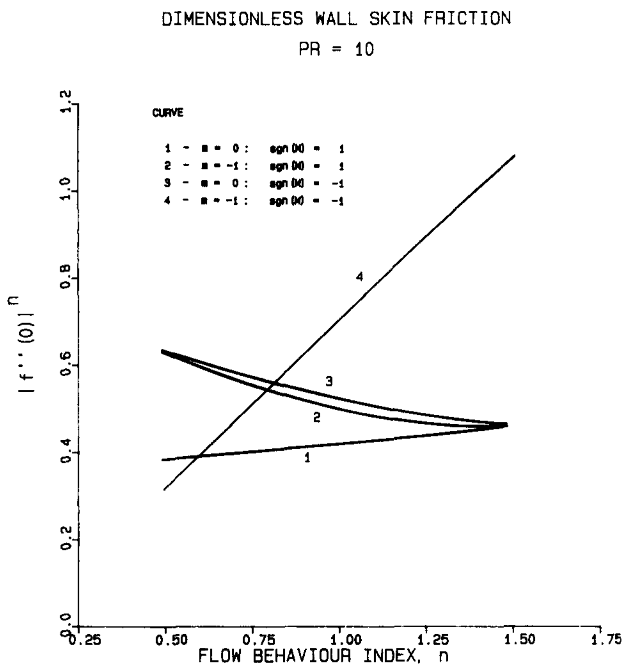

Figure 9 Dimensionless wall skin friction for various flow behavior indexes n ($Pr = 10$) for cases I, II, III, and IV

Finally, representative results for the dimensionless wall skin-friction coefficient $|f''(0)|^n$ in Equation 16 and dimensionless wall heat transfer rate $-|f''(0)|^{n-1}h'(0)$ in Equation 17 are presented in Figures 9 through 12. From Figures 9 and 10, we note that the friction factor decreases with n for cases ($m = -1$, $sgn(M) = 1$) and ($m = 0$, $sgn(M) = -1$). The reverse behavior is observed for cases ($m = 0$, $sgn(M) = 1$) and ($m = -1$, $sgn(M) = -1$). From Figures 11 and 12 we observe that the Nusselt number monotonically decreases with the flow behavior index, n, for all the cases considered. On the other hand, the impact of the generalized Prandtl number is more visible for the skin-friction coefficient than for the wall heat transfer rate. It is worth mentioning to this end that the information contained in Figures 9 to 12 should serve as a guide when one is trying to choose a non-Newtonian fluid for maximum heat transfer rate with minimum drag effects.

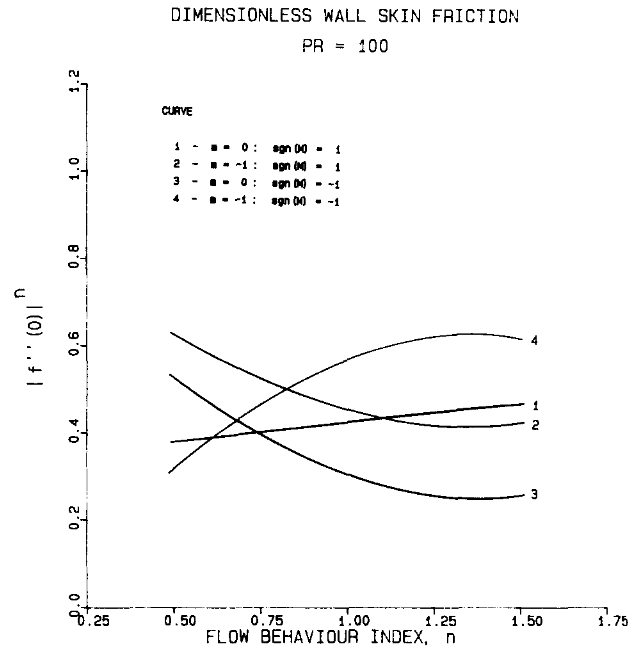

Figure 10 Dimensionless wall skin friction for various flow behavior indexes n ($Pr = 100$) for cases I, II, III, and IV

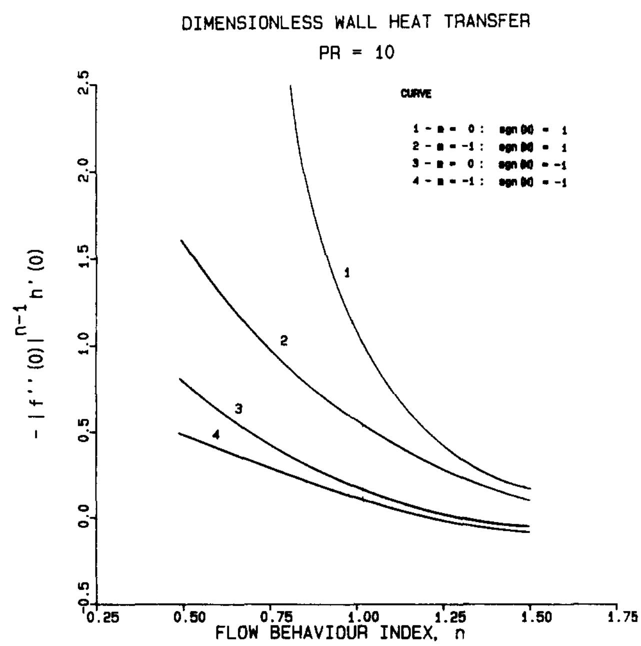

Figure 11 Dimensionless wall heat transfer rate for various flow behavior indexes n ($Pr = 10$) for cases I, II, III, and IV

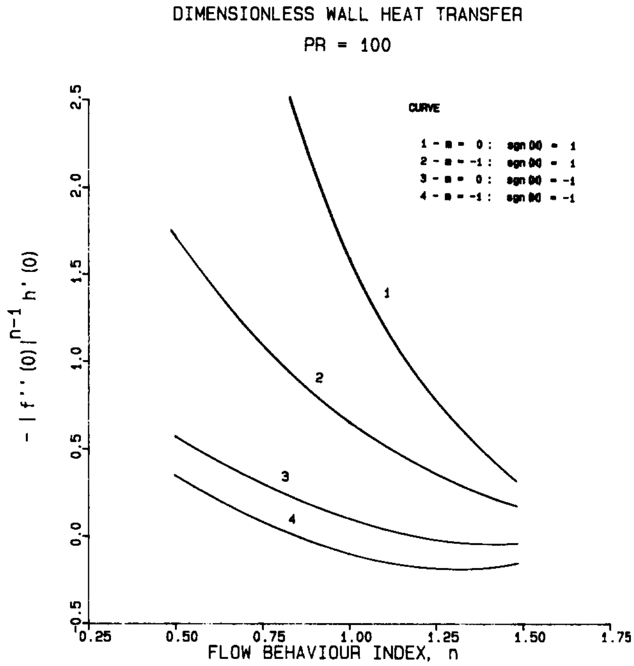

Figure 12 Dimensionless wall heat transfer rate for various flow behavior indexes n ($Pr = 100$) for cases I, II, III, and IV

Concluding remarks

The purpose of this article has been to determine numerically the similarity solutions for the problem of laminar natural convection from a vertical plate suspended in a quiescent thermally stratified power-law fluid within the boundary-layer scheme. New classes of solutions of the boundary-layer equations for two possible wall-temperature distributions were derived. As expected, it is found that both the momentum and thermal boundary-layer thickness increase with decreasing viscosity index n. However, the stratification parameter m has a pronounced effect on these boundary-layer thicknesses.

Solutions with $m = -1$ and $\mathrm{sgn}(M) = 1$ (unstable stratification) give higher velocity profiles than for $m = 0$ and $\mathrm{sgn}(M) = 1$ (non-stratified environment) when $Pr = 10$ and all values of the viscosity index n are considered (see Figure 1). But when $m = -1$ and $\mathrm{sgn}(M) = -1$ (stable stratification), the velocity profiles are lower for a pseudoplastic substance ($n = 0.5$) (see Figures 2 and 4). However, the reverse holds for a dilatant substance ($n = 1.5$) when Pr increases (see Figures 1 and 3). The skin-friction coefficient decreases for an unstably stratified environment and increases for a stable one. The wall heat transfer, on the contrary, decreases in both the stable and unstable environments. Both the Prandtl number and the stratification parameter have a more pronounced effect on the skin-friction coefficient than on the wall heat transfer.

Other, perhaps more important observations of this study include the flow and temperature reversals. Since no theoretical or experimental data exist for the problem of natural convection from an isothermal or nonisothermal flat plate placed in thermally stratified power-law fluids, it is not possible here to compare our results. Consequently, there is a definite need for more systematic studies for further evaluation and comparison for some flow and heat transfer situations.

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